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UNIFIED COMMON FIXED POINT THEOREM FOR MAPPINGS IN METRIC AND FUZZY METRIC SPACES

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ABSTRACT

"Fixed Point Theory" is one of the most fruitful and effective tools in analysis which has many applications within as well as outside mathematics. Fixed point theorem gives the conditions under which the involved mappings (single-valued and multi-valued) have unique solutions. During last 50 years or so, the theory of fixed points has come up as a powerful and important tool in the study of non- linear phenomena. Thus fixed point techniques are being applied in such diverse fields as Biology, Chemistry, Economics, Engineering, Game theory and physics.

Key words : Fixed Point Theory, mathematics

INTRODUCTION

In 1825 Cauchy did some fundamental work on existence of fixed point in differential equations. Fixed point theory find its origin in the use of successive approximations to establish the existence and uniqueness if solutions of differential and integral equations. The method is associated with celebrated mathematics such as Cauchy, Liouville, Lipschitz, Peano, Fredholm and Picard. However, it was the pioneering work of polish mathematician Stefen Banach [5] (1922) which credited him with placing underlying ideas into an abstract framework suitable for wide applications. In 1895 Poincare find fixed point application in the study of vector distribution on surface. He interpreted a vector distribution as a mapping of the surface to itself by translating a point as a vector based at that they found the isolated singularities of such distributions to which an index was assigned. These singularities are fixed point.

Fuzzy set theory offers us a new angle to observe and investigate the relation between set and their elements other than traditional "Black or White" way. It tells us besides "belonging to" and "not belonging to", other possibilities exist in the relation between an element and a set emerging in various practical processes. This point of view certainly offers us a new framework of set theory, and then in this new framework, we face the problems relating to mathematics, the study of which form the conditions of fuzzy mathematics.

Fuzzy theory holds that many things in life are matters of degree. A Black and White photo is not just Black and White; there are many levels of gray shades which can be observed in a typical picture. Computer scientists and engineers have long accepted this fact. As an example, a pixel can have a brightness value between 0 and 255. The value 0 stands for Black, 255 stands for White and every number between 0 and 255 stands for a certain gray level.

Let us consider the following sets

- The set of real numbers considerable larger than 10. (b) The set of all brilliant students of class B.Sc.
- The set of all tall players of Indian cricket team.
- The set of almost fine patient of hospital.
- The set of red tomatoes.

Such type of problems laid of foundation of the notion of fuzzy set. In the above example the words brilliant, tall, almost, red defines the set to be vague or fuzzy other than crisp (classical). This difficulty was overcome by the fuzzy concept. American Cyberneticist. L. A. Zadeh introduced Fuzzy set theory in (1965).

BROAD OUTLINES OF WORK

A result on the existence and uniqueness of common fixed point in metric spaces generally involves conditions on commutativity, continuity, and contraction along with a suitable condition on the containment of range one mapping into the range of other. Hence, one is always required to improve one or more of these conditions to prove a new fixed point theorem. In the recent past, several authors have contributed to the vigorous development of metric fixed point theory (e.g., [2, 6, 13, 14, 20-27, 33, 36-41, 43]).

The concept of a fuzzy set was introduced by Zadeh [44] in his seminal paper in 1965. In the last two decades, there has been a tremendous development and growth in fuzzy mathematics. In 1975, Kramosil and Michalek [28] introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces. Further, George and Veeramani [15] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [28] with a view to obtain a Hausdorff topology on it. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication, etc.

We propose to establish some new results for contraction mappings in metric spaces under weaker conditions. In 2002 Branciari [6] proved an integral analogue of Banach Contraction Mapping Principle.

We aim to establish new metrical common fixed point theorem for single valued mapping satisfying integral type contractive conditions.

Subsequently, there are a number of results proved for contraction mappings in fuzzy metric spaces ([1, 3, 4, 7-12, 16, 17, 19, 29-32, 34, 35, 42]). In our work propose to include some fixed point theorem in fuzzy metric spaces. Motivated by the work of Branciari [6], we propose to establish some fixed point results in integral analogue defined on fuzzy metric spaces.

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